OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

4723

Core Mathematics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Wednesday 20 January 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Turn over

1 Find $\int \frac{10}{(2x-7)^2}\,dx$. **[3]**

2 The angle θ is such that $0^\circ < \theta < 90^\circ$.

(i) Given that θ satisfies the equation $6\sin 2\theta = 5\cos\theta$, find the exact value of $\sin\theta$. **[3]**

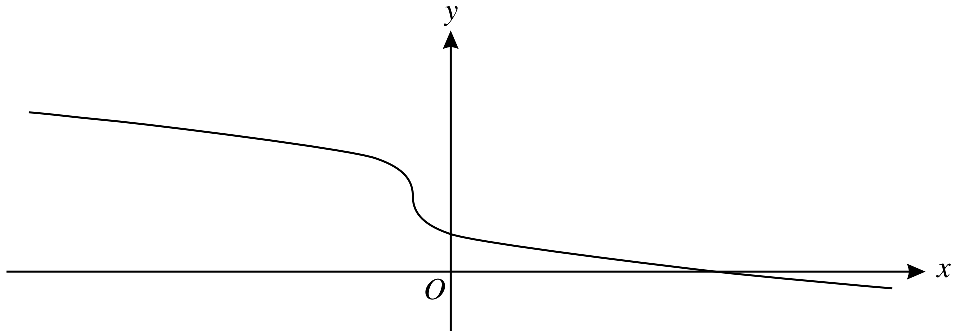
(ii) Given instead that θ satisfies the equation $8\cos\theta\,\mathrm{cosec}^2\,\theta = 3$, find the exact value of $\cos\theta$. **[5]**

3 **(i)** Find, in simplified form, the exact value of $\int_{10}^{20} \frac{60}{x}\,dx$. **[2]**

(ii) Use Simpson's rule with two strips to find an approximation to $\int_{10}^{20} \frac{60}{x}\,dx$. **[3]**

(iii) Use your answers to parts **(i)** and **(ii)** to show that $\ln 2 \approx \frac{25}{36}$. **[2]**

4



The function f is defined for all real values of x by

$$f(x) = 2 - \sqrt[3]{x+1}.$$

The diagram shows the graph of $y = f(x)$.

(i) Evaluate $ff(-126)$. **[2]**

(ii) Find the set of values of x for which $f(x) = |f(x)|$. **[2]**

(iii) Find an expression for $f^{-1}(x)$. **[3]**

(iv) State how the graphs of $y = f(x)$ and $y = f^{-1}(x)$ are related geometrically. **[1]**

5 The equation of a curve is $y = (x^2 + 1)^8$.

(i) Find an expression for $\frac{dy}{dx}$ and hence show that the only stationary point on the curve is the point for which $x = 0$. **[4]**

(ii) Find an expression for $\frac{d^2y}{dx^2}$ and hence find the value of $\frac{d^2y}{dx^2}$ at the stationary point. **[5]**

6 Given that

$$\int_0^{\ln 4} \left(ke^{3x} + (k-2)e^{-\frac{1}{2}x}\right) dx = 185,$$

find the value of the constant k. **[7]**

7 **(a)** Leaking oil is forming a circular patch on the surface of the sea. The area of the patch is increasing at a rate of 250 square metres per hour. Find the rate at which the radius of the patch is increasing at the instant when the area of the patch is 1900 square metres. Give your answer correct to 2 significant figures. **[4]**

(b) The mass of a substance is decreasing exponentially. Its mass now is 150 grams and its mass, m grams, at a time t years from now is given by

$$m = 150e^{-kt},$$

where k is a positive constant. Find, in terms of k, the number of years from now at which the mass will be decreasing at a rate of 3 grams per year. **[3]**

8 **(i)** The curve $y = \sqrt{x}$ can be transformed to the curve $y = \sqrt{2x+3}$ by means of a stretch parallel to the y-axis followed by a translation. State the scale factor of the stretch and give details of the translation. **[3]**

(ii) It is given that N is a positive integer. By sketching on a single diagram the graphs of $y = \sqrt{2x+3}$ and $y = \frac{N}{x^3}$, show that the equation

$$\sqrt{2x+3} = \frac{N}{x^3}$$

has exactly one real root. **[3]**

(iii) A sequence $x_1, x_2, x_3, \ldots$ has the property that

$$x_{n+1} = N^{\frac{1}{3}}(2x_n + 3)^{-\frac{1}{6}}.$$

For certain values of x_1 and N, it is given that the sequence converges to the root of the equation $\sqrt{2x+3} = \frac{N}{x^3}$.

(a) Find the value of the integer N for which the sequence converges to the value 1.9037 (correct to 4 decimal places). **[2]**

(b) Find the value of the integer N for which, correct to 4 decimal places, $x_3 = 2.6022$ and $x_4 = 2.6282$. **[3]**

[Question 9 is printed overleaf.]

9 The value of $\tan 10^\circ$ is denoted by p. Find, in terms of p, the value of

(i) $\tan 55^\circ$, **[3]**

(ii) $\tan 5^\circ$, **[4]**

(iii) $\tan \theta$, where θ satisfies the equation $3\sin(\theta + 10^\circ) = 7\cos(\theta - 10^\circ)$. **[5]**

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1		Obtain integral of form $k(2x-7)^{-1}$	M1		any constant k
		Obtain correct $-5(2x-7)^{-1}$	A1		or equiv
		Include … + c	B1	**3**	at least once; following any integral
				3	
2	**(i)**	Use $\sin 2\theta = 2\sin\theta\cos\theta$	B1		
		Attempt value of $\sin\theta$ from $k\sin\theta\cos\theta = 5\cos\theta$	M1		any constant k; or equiv
		Obtain $\frac{5}{12}$	A1	**3**	or exact equiv; ignore subsequent work
	(ii)	Use $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ or $\operatorname{cosec}^2\theta = 1+\cot^2\theta$	B1		or equiv
		Attempt to produce equation involving $\cos\theta$ only	M1		using $\sin^2\theta = \pm 1 \pm \cos^2\theta$ or equiv
		Obtain $3\cos^2\theta + 8\cos\theta - 3 = 0$	A1		or equiv
		Attempt solution of 3-term quadratic equation	M1		using formula or factorisation or equiv
		Obtain $\frac{1}{3}$ as only final value of $\cos\theta$	A1	**5**	or exact equiv; ignore subsequent work
				8	
3	**(i)**	Obtain or clearly imply $60\ln x$	B1		
		Obtain ($60\ln 20 - 60\ln 10$ and hence) $60\ln 2$	B1	**2**	with no error seen
	(ii)	Attempt calculation of form $k(y_0 + 4y_1 + y_2)$	M1		any constant k; using y-value attempts
		Identify k as $\frac{5}{3}$	A1		
		Obtain $\frac{5}{3}(6+4\times4+3)$ and hence $\frac{125}{3}$ or 41.7	A1	**3**	or equiv
	(iii)	Equate answers to parts **(i)** and **(ii)**	M1		provided $\ln 2$ involved
		Obtain $60\ln 2 = \frac{125}{3}$ and hence $\frac{25}{36}$	A1	**2**	AG; necessary detail required including clear use of an exact value from **(ii)**
				7	
4	**(i)**	Attempt correct process for composition	M1		numerical or algebraic
		Obtain (7 and hence) 0	A1	**2**	
	(ii)	Attempt to find x-intercept	M1		
		Obtain $x \le 7$	A1	**2**	or equiv; condone use of $<$
	(iii)	Attempt correct process for finding inverse	M1		
		Obtain $\pm(2-y)^3 - 1$ or $\pm(2-x)^3 - 1$	A1		
		Obtain correct $(2-x)^3 - 1$	A1	**3**	or equiv in terms of x
	(iv)	Refer to reflection in $y = x$	B1	**1**	or clear equiv
				8	

5	(i)	Obtain derivative of form $kx(x^2+1)^7$	M1		any constant k
		Obtain $16x(x^2+1)^7$	A1		or equiv
		Equate first derivative to 0 and confirm $x=0$ or substitute $x=0$ and verify first derivative zero	M1		AG; allow for deriv of form $kx(x^2+1)^7$
		Refer, in some way, to $x^2+1=0$ having no root	A1	**4**	or equiv
	(ii)	Attempt use of product rule	*M1		obtaining … + … form
		Obtain $16(x^2+1)^7$ + …	A1√		follow their $kx(x^2+1)^7$
		Obtain … + $224x^2(x^2+1)^6$	A1√		follow their $kx(x^2+1)^7$; or unsimplified equiv
		Substitute 0 in attempt at second derivative	M1		dep *M
		Obtain 16	A1	**5**	from second derivative which is correct at some point
				9	
6		Integrate e^{3x} to obtain $\frac{1}{3}e^{3x}$ or $e^{-\frac{1}{2}x}$ to obtain $-2e^{-\frac{1}{2}x}$	B1		or both
		Obtain indefinite integral of form $m_1e^{3x}+m_2e^{-\frac{1}{2}x}$	M1		any constants m_1 and m_2
		Obtain correct $\frac{1}{3}ke^{3x}-2(k-2)e^{-\frac{1}{2}x}$	A1		or equiv
		Obtain $e^{3\ln 4}=64$ or $e^{-\frac{1}{2}\ln 4}=\frac{1}{2}$	B1		or both
		Apply limits and equate to 185	M1		including substitution of lower limit
		Obtain $\frac{64}{3}k-(k-2)-\frac{1}{3}k+2(k-2)=185$	A1		or equiv
		Obtain $\frac{17}{2}$	A1	**7**	or equiv
				7	
7	(a)	<u>Either</u>: State or imply either $\frac{dA}{dr}=2\pi r$ or $\frac{dA}{dt}=250$	B1		or both
		Attempt manipulation of derivatives to find $\frac{dr}{dt}$	M1		using multiplication / division
		Obtain correct $\frac{250}{2\pi r}$	A1		or equiv
		Obtain 1.6	A1	**4**	or equiv; allow greater accuracy
		<u>Or</u>: Attempt to express r in terms of t	M1		using $A=250t$
		Obtain $r=\sqrt{\frac{250t}{\pi}}$	A1		or equiv
		Differentiate $kt^{\frac{1}{2}}$ to produce $\frac{1}{2}kt^{-\frac{1}{2}}$	M1		any constant k
		Substitute $t=7.6$ to obtain 1.6	A1	**(4)**	allow greater accuracy

	(b)	State $\frac{dm}{dt} = -150ke^{-kt}$	B1	
		Equate to $(\pm)3$ and attempt value for t	M1	using valid process; condone sign confusion
		Obtain $-\frac{1}{k}\ln(\frac{1}{50k})$ or $\frac{1}{k}\ln(50k)$ or $\frac{\ln 50 + \ln k}{k}$	A1 **3**	or equiv but with correct treatment of signs
			7	
8	**(i)**	State scale factor is $\sqrt{2}$	B1	allow 1.4
		State translation is in negative x-direction …	B1	or clear equiv
		… by $\frac{3}{2}$ units	B1 **3**	
	(ii)	Draw (more or less) correct sketch of $y = \sqrt{2x+3}$	B1	'starting' at point on negative x-axis
		Draw (more or less) correct sketch of $y = \frac{N}{x^3}$	B1	showing both branches
		Indicate one point of intersection	B1 **3**	with both sketches correct
		[SC: if neither sketch complete or correct but diagram correct for both in first quadrant B1]		
	(iii)	**(a)** Substitute 1.9037 into $x = N^{\frac{1}{3}}(2x+3)^{-\frac{1}{6}}$	M1	or into equation $\sqrt{2x+3} = \frac{N}{x^3}$; or equiv
		Obtain 18 or value rounding to 18	A1 **2**	with no error seen
		(b) State or imply $2.6282 = N^{\frac{1}{3}}(2\times 2.6022+3)^{-\frac{1}{6}}$	B1	
		Attempt solution for N	M1	using correct process
		Obtain 52	A1 **3**	concluding with integer value
			11	
9	**(i)**	Identify $\tan 55°$ as $\tan(45°+10°)$	B1	or equiv
		Use correct angle sum formula for $\tan(A+B)$	M1	or equiv
		Obtain $\frac{1+p}{1-p}$	A1 **3**	with $\tan 45°$ replaced by 1
	(ii)	Either: Attempt use of identity for $\tan 2A$	*M1	linking $10°$ and $5°$
		Obtain $p = \frac{2t}{1-t^2}$	A1	
		Attempt solution for t of quadratic equation	M1	dep *M
		Obtain $\frac{-1+\sqrt{1+p^2}}{p}$	A1 **4**	or equiv; and no second expression
		Or (1): Attempt expansion of $\tan(60°-55°)$	*M1	
		Obtain $\frac{\sqrt{3}-\frac{1+p}{1-p}}{1+\sqrt{3}\,\frac{1+p}{1-p}}$	A1√	follow their answer from **(i)**
		Attempt simplification to remove denominators	M1	dep *M
		Obtain $\frac{\sqrt{3}(1-p)-(1+p)}{1-p+\sqrt{3}(1+p)}$	A1 **(4)**	or equiv

Or (2):	State or imply $\tan 15° = 2 - \sqrt{3}$	B1	
	Attempt expansion of $\tan(15° - 10°)$	M1	with exact attempt for $\tan 15°$
	Obtain $\dfrac{2-\sqrt{3}-p}{1+p(2-\sqrt{3})}$	A2 **(4)**	
Or (3):	State or imply $\tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1}$	B1	or exact equiv
	Attempt expansion of $\tan(15° - 10°)$	M1	with exact attempt for $\tan 15°$
	Obtain $\dfrac{\sqrt{3}-1-p\sqrt{3}-p}{\sqrt{3}+1+p\sqrt{3}-p}$	A2 **(4)**	or equiv
Or (4):	Attempt expansion of $\tan(10° - 5°)$	*M1	
	Obtain $t = \dfrac{p-t}{1+pt}$	A1	
	Attempt solution for t of quadratic equation	M1	dep *M
	Obtain $\dfrac{-2+\sqrt{4+4p^2}}{2p}$	A1 **(4)**	or equiv; and no second expression

(iii)	Attempt expansion of both sides	M1	
	Obtain $3\sin\theta\cos 10° + 3\cos\theta\sin 10° = 7\cos\theta\cos 10° + 7\sin\theta\sin 10°$	A1	or equiv
	Attempt division throughout by $\cos\theta\cos 10°$	M1	or by $\cos\theta$ (or $\cos 10°$) only
	Obtain $3t + 3p = 7 + 7pt$	A1	or equiv
	Obtain $\dfrac{3p-7}{7p-3}$	A1 **5**	or equiv
		12	